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A Canticle on (4,0) Supergravity-Scalar Multiplet Systems for a "Cognoscente" [1]

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ABSTRACT

Extending prior investigations, we study three of the the four *distinct* minimal (4,0) scalar multiplets coupled to (4,0) supergravity. It is found that the scalar multiplets manifest their differences at the component level by possessing totally different couplings to the supergravity fields. Only the SM-I multiplet possesses a conformal coupling. For the remaining multiplets, terms linear in the world sheet curvature and/or SU(2) gauge field strengths are required to appear in the action by local supersymmetry.

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(I.) Introduction

One of the marvelous ways in which science differs from most human endeavors is that it is self-correcting. Furthermore, there are right answers and there are wrong answers. As such, the beliefs of even experts can be changed upon proof that their misconceptions are not grounded in reality. Some might say that parts of theoretical physics are not as well grounded in reality. However, even here we have rules of mathematical and logical consistency that act as a veto to the long term support of misconceptions and falsehoods.

Some time ago we argued that 2D representations of extended supersymmetry likely possess inherent ambiguities that permit the existence of many more distinct representations than one might naively guess. In the case of (4,0) supersymmetry we showed that this was precisely the case [1]. Remarkably enough if one considers the simplest (4,0) supersymmetric representation, the minimal scalar multiplet, it appears in four different varieties. Our observation acted as a generalization to that made by Witten in his work on ADHM non-linear σ-models [2] where it was proposed that two such multiplets (a (4,0) scalar multiplet and its twisted version) exist.

The work of [1] was partially inspired by Witten's remarkable proposal. Prior to the work on the ADHM σ-model, the only "twisting" known was related to 2D parity. In the language of conformal field theory, "twisting" is equivalent to the statement that the holomorphic and anti-holomorphic parts of a theory are not quite identical. However, in a (4,0) theory, there is no anti-holomorphic part of the theory to consider. So the question arose, "What is being twisted in Witten's ADHM models?" The answer provided by the work in [1] is that the twists in the ADHM σ-model take place with respect to the SU(2) group of conformal (4,0) supergravity.

Our (4,0) results obviously have implications for 2D (4,4) or $N = 4$ systems since (4,0) theories can be embedded into (4,4) theories. These implications were investigated in a study of 2D (4,4) hypermultiplets [3] that was completed some time ago. The result of that study was that eight distinct 2D, $N = 4$ hypermultiplets were found! This results goes against the general belief of some "experts" who have expressed unreasonable and unfounded skepticism. The criticism of our $N = 4$ results has been based on the naive statement that all of our "... distinct (4,0) scalar multiplet theories must be related by field redefinitions." Our response to this has been that the multiplets are not related by field re-definitions but are related by automorphisms of the supersymmetry parameters. The existence of such automorphisms has been known for over a decade. This is precisely the relation between 2D, $N = 2$ chiral multiplets and 2D, $N = 2$ twisted chiral multiplets [4].

When we describe two multiplets as being distinct or inequivalent, we mean that the set of all possible dynamics that can be described by use of one of the multiplets is distinct from that that can be described by use of the other one of the multiplets or by a linear combination of the two. The case of the 2D, $N = 2$ chiral multiplets versus 2D, $N = 2$ twisted chiral multiplets is a prototype example of this statement. The class of non-linear σ-models using 2D, $N = 2$ chiral multiplets must necessarily possess a Kähler geometry. Since a Kähler geometry is Riemannian, it has no torsion. On the other hand, the class of non-linear σ-models using 2D, $N = 2$ chiral multiplets and 2D, $N = 2$ twisted chiral multiplets can describe a complex geometry with torsion. Therefore, 2D, $N = 2$ chiral multiplets are distinct from 2D, $N = 2$ twisted chiral multiplets.

The simplest and most direct way to show that our (4,0) scalar multiplets are distinct is to explicitly demonstrate how the dynamics of the multiplets differ. There are many ways to do this. For example in [3] we showed how the massive dynamics of $N = 4$ hypermultiplets differ. One other demonstration of the different nature of the (4,0) hypermultiplets is to couple them to (4,0) supergravity. This will be the topic pursued in the present work.

There is a sense (explained in ref. [3]) in which all of the multiplets are "twisted" versions of one another. For example, SM-I and SM-III can be obtained one from the other by simply switching the Grassmann parity of all of the fields within a multiplet. We call this the "Klein flip."

(II.) The Varieties of Minimal (4,0) Scalar Multiplet Theory

In a previous work we have pointed out a generally unrecognized fact regarding (4,0) minimal irreducible scalar multiplet theories. Namely there are four distinct such theories. We denote these theories by SM-I, SM-II, SM-III and SM-IV. The field content of these are summarized in the following table.

Multiplet	Field Content
SM – I	$(\mathcal{A},\ \mathcal{B},\ \psi^{-i})$
SM – II	$(\phi,\ \phi_i{}^j,\ \lambda^-{}_i)$
SM – III	$(\mathcal{A}_i,\ \rho^-,\ \pi^-)$
SM – IV	$(\mathcal{B}_i,\ \psi^-,\ \psi_i^{-j})$

Table I

In this table each Latin letter index appended to a field denotes the defining representation of SU(2). All fields with two such indices are traceless. Each multiplet contains four bosons and four fermions. The bosons are $\mathcal{A}$, $\mathcal{B}$, ϕ, $\phi_i{}^j$, $\mathcal{A}_i$ and $\mathcal{B}_i$ and of these only ϕ and $\phi_i{}^j$ are real. Similarly, the fermions ψ^- and ψ_i^{-j} are real (Majorana).

(III.) Prepotential Formulation of Minimal (4,0) Scalar Multiplet Theory

The starting point of the manifestly supersymmetric quantization of a classical theory possessing supersymmetry is the construction of the description of that theory in terms of unconstrained superfields called pre-potentials. The main advantage of such a formulation is that it allows the powerful supergraph technique to be utilized in the exploration of quantum behavior of the classical theory. The most striking outcomes of such an approach are the derivation of non-renormalization theorems.

The (4,0) SM-I (scalar multiplet one) theory described in terms of constrained superfields is given by,

$$\begin{aligned} D_{+i}\mathcal{A} &= 2C_{ij}\psi^{-j} \quad , \quad & \overline{D}_+{}^i\mathcal{A} &= 0 \quad , \\ \overline{D}_+{}^i\mathcal{B} &= i2\psi^{-i} \quad , \quad & D_{+i}\mathcal{B} &= 0 \quad , \\ \overline{D}_+{}^i\psi^{-j} &= iC^{ij}\partial_{\ddagger}\mathcal{A} \quad , \quad & D_{+i}\psi^{-j} &= \delta_i{}^j\partial_{\ddagger}\mathcal{B} \quad . \end{aligned} \tag{1}$$

These superdifferential constraints can be solved explicitly in terms of a prepotential superfield ($P_{=-}{}^i$) that is subject to <u>no</u> differential constraints.

$$\begin{aligned} &\mathcal{A} \equiv \overline{D}_{\ddagger}^{\,2} D_{+i}P_{=-}{}^i \quad , \quad \mathcal{B} \equiv -iC_{ij}D_{\ddagger}^2\overline{D}_+{}^iP_{=-}{}^j \quad , \\ &\psi^{-i} \equiv -\tfrac{1}{2}C^{ij}D_{+j}\overline{D}_{\ddagger}^{\,2}D_{+k}P_{=-}{}^k \quad , \end{aligned} \tag{2}$$

where $D_{+i}D_{+j} \equiv C_{ij}D_{\ddagger}^2$ and $\overline{D}_+{}^i\overline{D}_+{}^j \equiv C^{ij}\overline{D}_{\ddagger}^{\,2}$. Using the algebra of the supercovariant derivatives it can be shown that the results in (1) follow now as simple consequences. The prepotential superfield is actually a gauge superfield. The quantities $\mathcal{A}$, $\mathcal{B}$ and ψ^{-i} are invariant under the gauge transformation given by

$$\delta_G P_{=-}{}^i = D_{+j}\Lambda_{==}^{(ij)} + \overline{D}_+{}^j\widehat{\Lambda}_{==j}{}^i \quad , \quad \widehat{\Lambda}_{==i}{}^i = 0 \quad . \tag{3}$$

The (4,0) SM-II (scalar multiplet two) theory described in terms of constrained superfields is given by,

$$\begin{aligned} &D_{+i}\,\phi = i\lambda^-{}_i \quad , \quad \phi = \phi^* \quad , \\ &D_{+i}\,\phi_j{}^k = 2\delta_i{}^k\lambda^-{}_j - \delta_j{}^k\lambda^-{}_i \quad , \quad \phi_i{}^j = (\phi_j{}^i)^* \quad , \quad \phi_i{}^i = 0 \quad , \\ &\overline{D}_+{}^i\,\lambda^-{}_j = \delta_j{}^i\partial_{\ddagger}\phi + i\partial_{\ddagger}\phi_j{}^i \quad , \quad D_{+i}\,\lambda^-{}_j = 0 \quad . \end{aligned} \tag{4}$$

These superdifferential constraints can be solved explicitly in terms of a prepotential superfield ($\Psi_{-\ j}$) that is subject to a chirality constraint $\overline{D}_+{}^i \Psi_{-\ j} = 0$,

$$
\begin{aligned}
\phi \ &\equiv \ -i[\ C^{ij} D_{+i} \Psi_{-\ j} \ + \ C_{ij} \overline{D}_+{}^i \overline{\Psi}_-{}^{\ j}\ ] \quad , \\
\phi_i{}^j \ &\equiv \ 2[\ C^{jk} D_{+k} \Psi_{-\ i} \ - \ C_{ik} \overline{D}_+{}^j \overline{\Psi}_-{}^{\ k}\ ] \ - \ \delta_i{}^j [\ C^{kl} D_{+k} \Psi_{-\ l} \ - \ C_{kl} \overline{D}_+{}^k \overline{\Psi}_-{}^{\ l}\ ] \quad , \\
\lambda^-{}_i \ &\equiv \ D_{\ddagger}^2 \Psi_{-\ i} \ - \ i2 C_{ij} \partial_{\ddagger} \overline{\Psi}_-{}^{\ j} \quad . \qquad (5)
\end{aligned}
$$

The field strength superfields above are invariant under the gauge transformation,

$$
\begin{aligned}
\delta_G \Psi_{-\ i} \ &= \ \overline{D}_{\ddagger}{}^2 [\ D_{+i} \Lambda_{=\!=} \ + \ i D_{+j} \widetilde{\Lambda}_{=\!= i}{}^j\ ] \quad , \quad \widetilde{\Lambda}_{=\!= i}{}^i \ = \ 0 \quad , \\
\Lambda_{=\!=} \ &= \ (\Lambda_{=\!=})^* \quad , \quad \widetilde{\Lambda}_{=\!= i}{}^j \ = \ (\widetilde{\Lambda}_{=\!= j}{}^i)^* \quad . \qquad (6)
\end{aligned}
$$

The (4,0) SM-III (scalar multiplet three) theory described in terms of constrained superfields is given by,

$$
\begin{aligned}
D_{+i} \mathcal{A}_j \ &= C_{ij} \pi^- \quad , \quad \overline{D}_+{}^i \mathcal{A}_j \ = \ \delta_j{}^i \rho^- \quad , \\
D_{+i} \rho^- \ &= i2\, \partial_{\ddagger} \mathcal{A}_i \quad , \quad \overline{D}_+{}^i \rho^- \ = \ 0 \quad , \qquad (7) \\
\overline{D}_+{}^i \pi^- \ &= i2\, C^{ij}\ \partial_{\ddagger} \mathcal{A}_j \quad , \quad D_{+i} \pi^- \ = \ 0 \quad .
\end{aligned}
$$

The solution to this set of superdifferential equations can be expressed in terms of two independent prepotential superfields (Σ_- and Υ_-) that each satisfies a chirality constraint ($\overline{D}_+{}^i \Sigma_- = 0$ and $\overline{D}_+{}^i \Upsilon_- = 0$). The explicit form of this solution is,

$$
\begin{aligned}
\mathcal{A}_i \ &= \ D_{+i} \Sigma_- \ + \ C_{ij} \overline{D}_+{}^j \overline{\Upsilon}_- \quad , \\
\pi^- \ &= \ D_{\ddagger}^2 \Sigma_- \ - \ i2\, \partial_{\ddagger} \overline{\Upsilon}_- \quad , \\
\rho^- \ &= \ \overline{D}_{\ddagger}{}^2 \overline{\Upsilon}_- \ + \ i2\, \partial_{\ddagger} \Sigma_- \quad . \qquad (8)
\end{aligned}
$$

These are invariant under the following gauge transformation,

$$
\delta_G \Sigma_- \ = \ \overline{D}_{\ddagger}{}^2 D_{+i} \Lambda_{=\!=}{}^i \quad , \quad \delta_G \Upsilon_- \ = \ -C^{ij} \overline{D}_{\ddagger}{}^2 D_{+i} \Lambda^*_{=\!=\ j} \quad . \qquad (9)
$$

The (4,0) SM-IV (scalar multiplet four) theory described in terms of constrained superfields is given by,

$$
\begin{aligned}
\overline{D}_+{}^i \mathcal{B}_j \ &= \delta_j{}^i \psi^- \ + \ i2\, \psi^-{}_j{}^i \quad , \quad D_{+i} \mathcal{B}_j \ = \ 0 \quad , \\
D_{+i} \psi^- \ &= i\, \partial_{\ddagger} \mathcal{B}_i \quad , \quad \psi^- \ = \ (\psi^-)^* \quad , \qquad (10) \\
D_{+i} \psi^-{}_j{}^k \ &= \ \delta_i{}^k \partial_{\ddagger} \mathcal{B}_j - \tfrac{1}{2} \delta_j{}^k\, \partial_{\ddagger} \mathcal{B}_i \quad , \quad \psi^-{}_i{}^j \ = \ (\psi^-{}_j{}^i)^* \quad .
\end{aligned}
$$

As above, these superdifferential constraints have an explicit solution given in terms of two independent prepotential superfields $U_{=-}$ and $V_{=-i}{}^{j}$. In order to write the solution to the constraints, it is convenient to define $S_{=-}$ and $T_{=-}{}_{i}{}^{j}$ as $S_{=-} \equiv U_{=-} + (U_{=-})^*$ and $T_{=-}{}_{i}{}^{j} \equiv V_{=-i}{}^{j} + (V_{=-i}{}^{j})^*$ so that $S_{=-} = -(S_{=-})^*$ and $T_{=-}{}_{i}{}^{j} = -(T_{=-}{}_{j}{}^{i})^*)$ are subject to <u>no</u> differential constraints.

$$
\begin{aligned}
\mathcal{B}_i ~&\equiv~ D_{\ddagger}^2[\, iC_{ij}\overline{D}_{+}{}^{j} S_{=-} ~+~ C_{jk}\overline{D}_{+}{}^{j} T_{=-}{}_{i}{}^{k}\,] \quad , \\
\psi^{-} ~&\equiv~ i\tfrac{1}{2}[\, C_{ij}\overline{D}_{+}{}^{i} D_{\ddagger}^2 \overline{D}_{+}{}^{j} S_{=-} ~+~ 2\,\partial_{\ddagger} D_{+i}\overline{D}_{+}{}^{j} T_{=-}{}_{j}{}^{i}\,] \quad , \\
\psi^{-}{}_{i}{}^{j} ~&\equiv~ \tfrac{1}{4}[\, (C_{ik}\overline{D}_{+}{}^{j} D_{\ddagger}^2 \overline{D}_{+}{}^{k} - C^{jk} D_{+i}\overline{D}_{\ddagger}{}^{2} D_{+k}) S_{=-}\,] \\
&\quad - i\tfrac{1}{4}[\, C_{kl}\overline{D}_{+}{}^{j} D_{\ddagger}^2 \overline{D}_{+}{}^{k} T_{=-}{}_{i}{}^{l} + C^{kl} D_{+i}\overline{D}_{\ddagger}{}^{2} D_{+k} T_{=-}{}_{l}{}^{j}\,] \quad . \qquad (11)
\end{aligned}
$$

These are invariant under a set of gauge variations given by,

$$
\begin{aligned}
\delta_G U_{=-} ~&=~ D_{+i}\widehat{\Lambda}_{==}{}^{i} \quad , \\
\delta_G V_{=-}{}_{i}{}^{j} ~&=~ -i\tfrac{2}{3}\, D_{+i}\widehat{\Lambda}_{==}{}^{j} ~+~ i\tfrac{1}{3}\delta_i{}^{j}\, D_{+k}\widehat{\Lambda}_{==}{}^{k} ~+~ C_{ip} D_{+q}\widehat{\Lambda}_{==}{}^{(jpq)} \quad . \qquad (12)
\end{aligned}
$$

This completes the unconstrained superfield description of the various scalar multiplets with manifest (4,0) supersymmetry. As can be seen, each of the scalar multiplets is described by a (set of) gauge prepotential superfields. These provide the fundamental superfields that can (in principle) be quantized and used to generate supergraph rules. One other interesting observation is that the 2D Lorentz representation of the gauge parameter superfield for <u>all</u> the scalar multiplets is the same. All such superfields transform as the minus two representation of the 2D lorentz group.

For the SM-I, SM-II, SM-III and SM-IV models, the free actions are obtained from the following respective superspace expressions,

$$
\begin{aligned}
\mathcal{S}_{\rm SM-I} ~&=~ \Big[\int d^2\sigma\, d^2\zeta^{\ddagger}\, [\, -i\tfrac{1}{4}\overline{\mathcal{B}}\partial_{=}\mathcal{A}\,] ~+~ {\rm h.c.} \Big] \quad , \\
\mathcal{S}_{\rm SM-II} ~&=~ \Big[\int d^2\sigma\, d^2\zeta^{\ddagger}\, [\, -\tfrac{1}{2}\Psi_{-i}\partial_{=}\overline{\lambda}^{-i}\,] ~+~ {\rm h.c.} \Big] \quad , \\
\mathcal{S}_{\rm SM-III} ~&=~ \Big[\int d^2\sigma\, d^2\zeta^{\ddagger}\, [\, -\tfrac{1}{8}\Sigma_{-}\partial_{=}\overline{\pi}^{-} ~+~ \tfrac{1}{8}\Upsilon_{-}\partial_{=}\rho^{-}\,] ~+~ {\rm h.c.} \Big] \quad , \\
\mathcal{S}_{\rm SM-IV} ~&=~ \Big[\int d^2\sigma\, d^2\zeta^{\ddagger}\, [\, -i\tfrac{1}{4}C_{ij}\overline{\mathcal{B}}^{i}\partial_{=}\overline{\mathcal{B}}^{j}\,] ~+~ {\rm h.c.} \Big] \quad . \qquad (13)
\end{aligned}
$$

Now the critical feature about these expressions is that in order to write the actions for SM-II and SM-III, we had to explicitly express them in terms of prepotentials. This is vastly different from the SM-I and SM-IV theory where their chiral actions

were totally expressible *solely* in terms of the field strength superfields. We know that the actions for SM-II and SM-III above only involve the component fields contained in the field strength superfields because these actions are *gauge invariant* with respect to the prepotential gauge transformations.

(IV.) (4,0) Supergravity Theory and Superstrings

The supergeometry of (p, 0) supergravity has been known for some time [5]. It is simple to specialize to the case of p = 4. There is a supergravity covariant derivatives $\nabla_A \equiv (\nabla_{+i},\, \overline{\nabla}_+{}^i,\, \nabla_{\mp},\, \nabla_{=})$ that can be expanded over a supervielbein $(E_A{}^M)$, Lorentz spin-connection (ω_A) and SU(2) gauge connection $(\mathcal{A}_{A\,i}{}^j)$,

$$\nabla_A \;=\; E_A{}^M \mathrm{D}_M \;+\; \omega_A \mathcal{M} \;+\; i\,\mathcal{A}_{A\,i}{}^j \mathcal{Y}_j{}^i \quad . \tag{14}$$

Above D_M denotes the flat space fermi and bose derivatives $\mathrm{D}_M \equiv (\overline{\mathrm{D}}_{+i},\, \mathrm{D}_+{}^i,\, \partial_{\mp},\, \partial_{=})$. Similarly, $\mathcal{M}$ and $\mathcal{Y}_i{}^j$ denotes the Lorentz and SU(2) generators respectively. These act on ∇_A as

$$[\,\mathcal{M}, \nabla_{+i}\,] \;=\; \tfrac{1}{2}\,\nabla_{+i} \quad , \quad [\,\mathcal{M}, \overline{\nabla}_+{}^i\,] \;=\; \tfrac{1}{2}\,\overline{\nabla}_+{}^i \quad , \quad [\,\mathcal{M}, \nabla_{\mp}\,] \;=\; \nabla_{\mp} \quad . \tag{15}$$

$$[\,\mathcal{Y}_j{}^k, \nabla_{+i}\,] \;=\; \delta_i{}^k \nabla_{+j} \;-\; \tfrac{1}{2}\,\delta_j{}^k \nabla_{+i} \quad , \quad [\,\mathcal{Y}_j{}^k, \overline{\nabla}_+{}^i\,] \;=\; -\delta_j{}^i \overline{\nabla}_+{}^k \;+\; \tfrac{1}{2}\,\delta_j{}^k \overline{\nabla}_+{}^i \quad ,$$

$$[\,\mathcal{M}, \nabla_{=}\,] \;=\; -\,\nabla_{=} \quad , \quad [\,\mathcal{Y}_j{}^k, \nabla_{\mp}\,] \;=\; 0 \quad , \quad [\,\mathcal{Y}_j{}^k, \nabla_{=}\,] \;=\; 0 \quad .$$

The covariant derivatives have a commutator algebra that takes the form

$$[\;\nabla_{+i}\,,\,\nabla_{+j}\;\} \;=\; 0 \quad , \quad [\;\nabla_{+i}\,,\,\overline{\nabla}_+{}^j\;\} \;=\; i2\delta_i{}^j\,\nabla_{\mp} \quad , \quad [\;\nabla_{+i}\,,\,\nabla_{\mp}\;\} \;=\; 0 \quad ,$$

$$[\;\nabla_{+i}\,,\,\nabla_{=}\;\} \;=\; -i\,[\;\overline{\Sigma}^+{}_i \mathcal{M} \;-\; \overline{\Sigma}^+{}_j \mathcal{Y}_i{}^j\;] \quad ,$$

$$[\;\nabla_{\mp},\,\nabla_{=}\;\} \;=\; -\tfrac{1}{2}\,[\;\Sigma^{+i}\,\nabla_{+i} \;+\; \overline{\Sigma}^+{}_i\,\overline{\nabla}_+{}^i \;+\; \mathcal{R}\mathcal{M} \;+\; i\mathcal{F}_i{}^j \mathcal{Y}_j{}^i\;] \quad . \tag{16}$$

These lead to a set of Bianchi identities that are solved if

$$\overline{\nabla}_+{}^i\,\Sigma^{+j} \;=\; 0 \quad , \quad \nabla_{+i}\,\Sigma^{+j} \;=\; \tfrac{1}{2}\,\delta_i{}^j \mathcal{R} \;+\; i\,\mathcal{F}_i{}^j \quad ,$$

$$\nabla_{+i}\,\mathcal{R} \;=\; i\,2\nabla_{\mp}\overline{\Sigma}^+{}_i \quad , \quad \nabla_{+i}\,\mathcal{F}_j{}^k \;=\; -\,2\delta_i{}^k \nabla_{\mp}\overline{\Sigma}^+{}_j \;+\; \delta_j{}^k \nabla_{\mp}\overline{\Sigma}^+{}_i \quad . \tag{17}$$

The component fields of the (4,0) supergravity multiplet are $e_a{}^m$ (a zweibein), $\psi_a{}^{+\,i}$ (SU(2) doublet gravitini) and $A_{a\,i}{}^j$ (gauge SU(2) triplet of auxiliary fields). The supersymmetry variations of these may be chosen to take the forms,

$$\delta_Q e_{\mp}{}^m \;=\; \delta_Q A_{\mp\,i}{}^j \;=\; 0 \quad , \quad \delta_Q \psi_m{}^{+\,i} \;=\; \mathcal{D}_m \epsilon^{+\,i} \quad ,$$

$$\delta_Q e_{=}{}^m \;=\; -i2g^{mn}[\,\overline{\epsilon}^+{}_i \psi_n{}^{+\,i} \;+\; \epsilon^{+\,i}\overline{\psi}_n{}^+{}_i\,] \quad ,$$

$$\delta_Q A_{=\,i}{}^j \;=\; \Big[2\,[\,\epsilon^{+\,j}\overline{\psi}_{\ddagger,\,=}{}^{+}{}_i \;-\; \tfrac{1}{2}\delta_i{}^j\epsilon^{+\,k}\overline{\psi}_{\ddagger,\,=}{}^{+}{}_k\,] \;+\; \text{h.c.}\Big] \quad , \tag{18}$$

where $g^{mn} \equiv [\, e_{\ddagger}{}^n e_{=}{}^m \;+\; e_{\ddagger}{}^n e_{=}{}^m\,]$.

We next turn to the problem of finding (4,0) locally supersymmetric actions. If $\mathcal{L}_{=}$ is a chiral Lagrangian ($\overline{\nabla}_+{}^i\mathcal{L}_{=} = 0$) and $\overline{\mathcal{L}}_{=}$ is an anti-chiral Lagrangian ($\nabla_{+i}\overline{\mathcal{L}}_{=} = 0$), then component actions are derivable from

$$\begin{aligned} \int d^2\sigma d^2\zeta^{\ddagger}\; \mathcal{E}^{-1}\mathcal{L}_{=}\,| \quad &\equiv \quad i\int d^2\sigma\,\big[\,\tfrac{1}{2}\,e^{-1}\,C^{ij}\,\big(\nabla_{+i} + i4\,e\,\overline{\psi}_{\ddagger}{}^{+}{}_i\big)\big]\,\nabla_{+j}\mathcal{L}_{=}\,| \quad , \\ \int d^2\sigma d^2\overline{\zeta}^{\ddagger}\;\overline{\mathcal{E}}^{-1}\,\overline{\mathcal{L}}_{=}\,| \quad &\equiv \quad i\int d^2\sigma\,\big[\,\tfrac{1}{2}\,e^{-1}\,C_{ij}\,\big(\overline{\nabla}_+{}^i + i4\,e\,\psi_{\ddagger}{}^{+i}\big)\big]\,\overline{\nabla}_+{}^j\overline{\mathcal{L}}_{=}\,| \quad . \end{aligned} \tag{19}$$

For a general Lagrangian $\mathcal{L}_{==}$, the component action follows from

$$\begin{aligned} \int d^2\sigma\, d^2\zeta^{\ddagger}\, d^2\overline{\zeta}^{\ddagger}\; E^{-1}\mathcal{L}_{==} \quad \equiv \quad &\tfrac{1}{2}\int d^2\sigma\, d^2\zeta^{\ddagger}\,\mathcal{E}^{-1}\,\big[\,\tfrac{1}{2}C_{ij}\,\overline{\nabla}_+{}^i\,\overline{\nabla}_+{}^j\,\big]\;\mathcal{L}_{==}\,| \quad + \\ &\tfrac{1}{2}\int d^2\sigma\, d^2\overline{\zeta}^{\ddagger}\;\overline{\mathcal{E}}^{-1}\,\big[\,\tfrac{1}{2}C^{ij}\,\nabla_{+i}\nabla_{+j}\,\big]\,\mathcal{L}_{==}\,| \quad . \end{aligned} \tag{20}$$

Thus, we find that the density multiplet formulae provide a simple prescription for deriving locally (4,0) supersymmetrically invariant component actions from superspace actions.

It is well known that the critical dimension of (4,0) strings is such that classical conformal invariance does not survive quantization. Even so, we now have a lot of experiences to indicate that there are still interesting phenomena occurring within such theories. The fact that there are four different (4,0) scalar multiplets adds an extra twist...there are four candidates from which to start. These are the local versions of the actions in (13)

$$\mathcal{S}_{\rm SM-I} \;=\; \Big[\;\int d^2\sigma\, d^2\zeta^{\ddagger}\,\mathcal{E}^{-1}\;\{-\,i\tfrac{1}{4}\overline{\mathcal{B}}\,\nabla_{=}\mathcal{A}\,\} \;+\; \text{h.c.}\Big] \quad , \tag{21}$$

$$\mathcal{S}_{\rm SM-II} \;=\; \Big[\;\int d^2\sigma\, d^2\zeta^{\ddagger}\,\mathcal{E}^{-1}\;[\,-\,\tfrac{1}{2}\Psi_{-i}\{\,\nabla_{=}\overline{\lambda}^{-i} \;+\; \tfrac{1}{2}(\Sigma^{+\,i}\phi \;+\; i\Sigma^{+\,j}\phi_j{}^i)\,\}\,] \;+\; \text{h.c.}\Big] \quad , \tag{22}$$

$$\begin{aligned} \mathcal{S}_{\rm SM-III} \;=\; \Big[\;\int d^2\sigma\, d^2\zeta^{\ddagger}\,\mathcal{E}^{-1}\;[\;-\;\tfrac{1}{8}\Sigma_{-}\{\,\nabla_{=}\overline{\pi}^{-} \;+\; i\tfrac{1}{8}C_{ij}\Sigma^{+\,i}\overline{\mathcal{A}}^j\,\} \\ +\;\tfrac{1}{8}\Upsilon_{-}\{\,\nabla_{=}\rho^{-} \;-\; i\tfrac{1}{2}\Sigma^{+\,i}\mathcal{A}_i\,\}\,] \;+\; \text{h.c.}\Big] \quad . \end{aligned} \tag{23}$$

Here it is appropriate to make comments on these action formulae as well as that of SM-IV. The actions above are found by beginning with the rigid results and demanding the existence of their local extensions. In particular, the chirality requirement of the integrands demands the appearance of the (4,0) supergravity field

strength supertensor. We thus find non-minimal coupling to the supergravity fields. These are explicitly seen for SM-II and SM-III theories. However, no non-minimal coupling is required for SM-I. These results illustrate the “unknown” theorem that we have noted several times previously [3], [6], [7]. Namely, the result that the coupling to SM-I is minimal corresponds to the component level statement that the spin-0 fields in the SM-I multiplet are singlets under the (4,0) superholonomy group. Note that the non-minimal coupling is such that only the supergravity-SM-I system possesses the full (4,0) superconformal invariance required of a string theory!

The reader will note that we have not presented a local extension for the SM-IV theory. The reason for this is that at present there are still some aspects of this theory that are being studied further. We hope to report on this in the near future.

(V.) (4,0) Supergravity Coupled to Minimal (4,0) Scalar Multiplets: Component Results

Having derived the superspace form of the local versions of three of our four multiplets, we wish to investigate the component results that follow as consequences. The distinctiveness of each multiplet will be crystal clear as a result. All of our results below follow from the straightforward application of the density projectors developed in the previous section.

The SM-I multiplet has the following locally supersymmetrically invariant action,

$$
\begin{aligned}
\mathcal{S}_{\rm SM-I} ~=~ \int d^2\sigma\, e^{-1} \Big[~ & \tfrac{1}{2} g^{mn} \{ (\partial_m \overline{\mathcal{A}})(\partial_n \mathcal{A}) ~+~ (\partial_m \overline{\mathcal{B}})(\partial_n \mathcal{B}) \} \\
& - i \{ \overline{\psi}{}^{-}{}_i \mathcal{D}_{=} \psi^{-i} - (\mathcal{D}_{=} \overline{\psi}{}^{-}{}_i) \psi^{-i} \} \\
& + 2 (\mathcal{D}_{\ddagger} \overline{\mathcal{A}}) \{ C_{ij} \psi_{=}{}^{+i} \psi^{-j} \} - 2 (\mathcal{D}_{\ddagger} \mathcal{A}) \{ C^{ij} \overline{\psi}_{=}{}^{+}{}_i \overline{\psi}{}^{-}{}_j \} \\
& + 2i (\mathcal{D}_{\ddagger} \overline{\mathcal{B}}) \{ \overline{\psi}_{=}{}^{+}{}_i \psi^{-i} \} + 2i (\mathcal{D}_{\ddagger} \mathcal{B}) \{ \psi_{=}{}^{+i} \overline{\psi}{}^{-}{}_i \} \\
& - 2 \{ \psi_{\ddagger}{}^{+i} \overline{\psi}{}^{-}{}_i \} \{ \overline{\psi}_{=}{}^{+}{}_j \psi^{-j} \} - 2 \{ \psi_{=}{}^{+i} \overline{\psi}{}^{-}{}_i \} \{ \overline{\psi}_{\ddagger}{}^{+}{}_j \psi^{-j} \} \\
& - 2 \{ C_{ij} \psi_{\ddagger}{}^{+i} \psi^{-j} \} \{ C^{kl} \overline{\psi}_{=}{}^{+}{}_k \overline{\psi}{}^{-}{}_l \} \\
& - 2 \{ C_{ij} \psi_{=}{}^{+i} \psi^{-j} \} \{ C^{kl} \overline{\psi}_{\ddagger}{}^{+}{}_k \overline{\psi}{}^{-}{}_l \} ~ \Big] ~~ .
\end{aligned}
\tag{24}
$$

In the case of the SM-II theory the component result takes the form,

$$
\begin{aligned}
\mathcal{S}_{\rm SM-II} ~=~ \int d^2\sigma\, e^{-1} \Big[~ & \tfrac{1}{2} (\mathcal{D}_{\ddagger} \phi)(\mathcal{D}_{=} \phi) + \tfrac{1}{4} (\mathcal{D}_{\ddagger} \phi_i{}^j)(\mathcal{D}_{=} \phi_j{}^i) \\
& - \tfrac{i}{2} \{ \overline{\lambda}{}^{-i} \mathcal{D}_{=} \lambda^{-}{}_i - (\mathcal{D}_{=} \overline{\lambda}{}^{-i}) \lambda^{-}{}_i \} + \tfrac{1}{4} \phi_i{}^j \mathcal{F}_j{}^i \phi \\
& - \tfrac{1}{4} \{ \phi^2 - \tfrac{1}{2} \phi_i{}^j \phi_j{}^i \} \{ \tfrac{1}{2} \mathcal{R} - i \psi_{\ddagger}{}^{+k} \overline{\Sigma}{}^{+}{}_k - i \overline{\psi}_{\ddagger}{}^{+}{}_k \Sigma^{+k} \} \\
& - \{ \lambda^{-}{}_i \psi_{=}{}^{+j} - \overline{\lambda}{}^{-j} \overline{\psi}_{=}{}^{+}{}_i \} \mathcal{D}_{\ddagger} \phi_j{}^i - \tfrac{1}{2} \{ \Sigma^{+j} \lambda^{-}{}_i - \overline{\Sigma}{}^{+}{}_i \overline{\lambda}{}^{-j} \} \phi_j{}^i
\end{aligned}
$$

$$
\begin{aligned}
&- i\{\lambda^-{}_i\psi_{=}{}^{+i} + \overline{\lambda}^{-i}\overline{\psi}_{=}{}^+{}_i\}\mathcal{D}_{\ddagger}\phi + \tfrac{i}{2}\{\Sigma^{+i}\lambda^-{}_i + \overline{\Sigma}^+{}_i\overline{\lambda}^{-i}\}\phi \\
&+ \{C^{ij}\lambda^-{}_i\lambda^-{}_j\}\{C_{kl}\psi_{=}{}^{+k}\psi_{\ddagger}{}^{+l}\} \\
&+ \{C_{ij}\overline{\lambda}^{-i}\overline{\lambda}^{-j}\}\{C^{kl}\overline{\psi}_{=}{}^+{}_k\overline{\psi}_{\ddagger}{}^+{}_l\} \\
&- \{\overline{\lambda}^{-i}\lambda^-{}_i\}\{\psi_{=}{}^{+j}\overline{\psi}_{\ddagger}{}^+{}_j + \psi_{\ddagger}{}^{+j}\overline{\psi}_{=}{}^+{}_j\} \ \Big] \quad .
\end{aligned}
\tag{25}
$$

In the case of the SM-III theory the component result takes the form,

$$
\begin{aligned}
\mathcal{S}_{\rm SM-III} = \int d^2\sigma \, e^{-1} \, \Big[& \tfrac{1}{4}g^{mn}(\mathcal{D}_m\overline{\mathcal{A}}^i)(\mathcal{D}_n\mathcal{A}_i) \\
& - \tfrac{i}{8}\{\overline{\pi}^-\mathcal{D}_{=}\pi^- - (\mathcal{D}_{=}\overline{\pi}^-)\pi^-\} - \tfrac{i}{8}\{\overline{\rho}^-\mathcal{D}_{=}\rho^- - (\mathcal{D}_{=}\overline{\rho}^-)\rho^-\} \\
& - \tfrac{1}{2}\{\psi_{=}{}^{+i}\overline{\rho}^- - C^{ij}\overline{\psi}_{=}{}^+{}_j\overline{\pi}^-\}\mathcal{D}_{\ddagger}\mathcal{A}_i \\
& + \tfrac{1}{2}\{\overline{\psi}_{=}{}^+{}_i\rho^- - C_{ij}\psi_{=}{}^{+j}\pi^-\}\mathcal{D}_{\ddagger}\overline{\mathcal{A}}^i \\
& + \tfrac{1}{8}\overline{\mathcal{A}}^i\mathcal{A}_i\{\tfrac{1}{2}\mathcal{R} - i\psi_{\ddagger}{}^{+j}\overline{\Sigma}^+{}_j - i\overline{\psi}_{\ddagger}{}^+{}_j\Sigma^{+j}\} \\
& + \tfrac{1}{8}C_{ij}\overline{\mathcal{A}}^i\Sigma^{+j}\pi^- - \tfrac{1}{8}C^{ij}\mathcal{A}_i\overline{\Sigma}^+{}_j\overline{\pi}^- + \tfrac{1}{8}\Sigma^{+i}\mathcal{A}_i\overline{\rho}^- - \tfrac{1}{8}\overline{\Sigma}^+{}_i\overline{\mathcal{A}}^i\rho^- \\
& - \tfrac{1}{4}\{\psi_{=}{}^{+i}\overline{\psi}_{\ddagger}{}^+{}_i + \psi_{\ddagger}{}^{+i}\overline{\psi}_{=}{}^+{}_i\}\{\overline{\pi}^-\pi^- - \overline{\rho}^-\rho^-\} \\
& + \tfrac{1}{2}\{C_{ij}\psi_{=}{}^{+i}\psi_{\ddagger}{}^{+j}\}\{\pi^-\overline{\rho}^-\} \\
& + \tfrac{1}{2}\{C^{ij}\overline{\psi}_{=}{}^+{}_i\overline{\psi}_{\ddagger}{}^+{}_j\}\{\overline{\pi}^-\rho^-\} \ \Big] \quad .
\end{aligned}
\tag{26}
$$

In order to simplify the subsequent discussion, let us set all purely fermionic terms to zero to obtain

$$
\mathcal{S}_{\rm SM-I} = \int d^2\sigma \, e^{-1} \, \Big[\, \tfrac{1}{2}g^{mn}\{ \, (\partial_m\overline{\mathcal{A}})(\partial_n\mathcal{A}) \, + \, (\partial_m\overline{\mathcal{B}})(\partial_n\mathcal{B})\} \, \Big] \quad , \tag{27}
$$

$$
\begin{aligned}
\mathcal{S}_{\rm SM-II} = \int d^2\sigma \, e^{-1} \, \Big[\, & \tfrac{1}{2}(\mathcal{D}_{\ddagger}\phi)(\mathcal{D}_{=}\phi) + \tfrac{1}{4}(\mathcal{D}_{\ddagger}\phi_i{}^j)(\mathcal{D}_{=}\phi_j{}^i) \\
& + \tfrac{1}{4}\phi_i{}^j\mathcal{F}_j{}^i\phi - \tfrac{1}{8}\,\{\phi^2 - \tfrac{1}{2}\phi_i{}^j\phi_j{}^i\}\mathcal{R} \ \Big] \quad ,
\end{aligned}
\tag{28}
$$

$$
\mathcal{S}_{\rm SM-III} = \int d^2\sigma \, e^{-1} \, \Big[\tfrac{1}{4}g^{mn}(\mathcal{D}_m\overline{\mathcal{A}}^i)(\mathcal{D}_n\mathcal{A}_i) \, + \, \tfrac{1}{16}\overline{\mathcal{A}}^i\mathcal{A}_i\mathcal{R} \, \Big] \quad . \tag{29}
$$

Note that SM-I possesses a completely conformal coupling of the world sheet zweibein to the spin-0 fields of the matter multiplet. For SM-II, the world sheet curvature ($\mathcal{R}$) as well as the SU(2) field strength ($\mathcal{F}_i{}^j$) are both coupled to the spin-0 fields. This is in addition to the implicit minimal SU(2) gauge field coupling inside the covariant derivatives. Finally for SM-III we see only the world sheet curvature as an explicit coupling to the spin-0 fields as well the implicit coupling to the SU(2) gauge fields via the covariant derivatives.

(VI.) Discussion

One of the interesting consequences of our study of local (4,0) actions is that it permits us to answer questions that were raised in the immediate past. It was noticed that the phenomenon of a multiplicity of scalar multiplets also occurs in full (4,4) theory. This led to our asking the natural questions [8], "Why are there so many $N = 4$ superstrings?" and "How many $N = 4$ superstrings exist?" In fact, we found evidence of <u>eight</u> 2D (4,4) hypermultiplets [3]. On the basis of our study of (4,0) models, the answer to these questions are, "Parity and four, respectively" In the notation of [3] these $N = 4$ superstrings are based on the $4s^+$, $3s^+s^-$ and $2s^+2s^-$ hypermultiplets[2]. The concept of distinct extended superstrings for a fixed value of N may be new to some of our readers. So it may useful to review the first discovery of this phenomenon in a simpler context and use some concepts from superconformal field theory.

A number of years ago [9] it was pointed out that within the context of $N = 2$ superstrings, there <u>must</u> exist a minimum of <u>three</u> <u>distinct</u> theories! This was based on the fact that more than one type of $N = 2$ scalar multiplet was known to exist. There is the standard $2D$, $N = 2$ chiral scalar multiplet as well as the $2D$, $N = 2$ twisted chiral scalar multiplet. Either of these two scalar multiplets can be used to write anomaly-free $2D$, $N = 2$ superstrings and there are three possible ways to carry out such a construction. We shall call these the C^2, CT and T^2 $N = 2$ superstrings. The existence of both the chiral scalar multiplet and the twisted chiral scalar multiplet are a reflection of the fact that both (c,c) and (a,c) rings exist within $2D$, $N = 2$ superconformal field theory. The former correspond to chiral superfields while the latter correspond to twisted chiral superfields. Thus, in the construction of $2D$, $N = 2$ superstrings, there is one version where the matter superfields possess mirror symmetry (the CT $N = 2$ superstring) if we neglect supergravity and two versions that are mirror asymmetric (the C^2 and T^2 $N = 2$ superstrings). If we neglect supergravity, these latter two theories are the "mirror reflections" of each other. A fundamental difference between a chiral and twisted chiral multiplet is that the spin-0 states of the former have the same parity while those of the latter have opposite parities.

Returning now to the $N = 4$ case, we obtain the number four due to the following implication of our work. For the (4,0) superstrings, we saw that in coupling the scalar multiplets to supergravity a very interesting phenomenon occurred. Namely, whenever the scalar fields transformed non-trivially under the SU(2) of (4,0) super-

[2] We have noted previously that there is a two-fold degeneracy in the $2s^+2s^-$ case.

gravity, the locally supersymmetric action demanded the presence of non-conformal couplings in the world sheet action! In other words, terms linear in the world sheet curvature and quadratic in spin-0 fields are present unless the spin-0 fields were SU(2) singlets. This same phenomenon must occur in the full (4,4) candidate superstring actions! It is only in the case of the $4s^+$, $3s^+s^-$ and $2s^+2s^-$ 2D (4,4) hypermultiplets that the spin-0 fields are in the trivial representation of the SU(2) that is gauged by (4,4) supergravity! Thus a classification of the presently known distinct $2D$, N = 4 superstrings consists of the $4s^+$, $3s^+s^-$, $2s^+2s^-$A and $2s^+2s^-$B superstrings. In otherword, $N = 4$ superstrings exist with either zero, one or two psuedo-scalar spin-0 fields replacing the usual scalar spin-0 fields. These are the direct generalizations of the analogous $N = 2$ results (i.e. all of these are connected by different parity twists) and show that there is intrinsic non-uniqueness in $N = 4$ superstring theory (exactly like $N = 2$ theory) contrary to other suggestions [10]. Stated another way, it is not N = 4 superstrings that are unique but instead it is the (4,0) superstring that presently seems unique. One final point is that the existence of both 2D, $N = 2$ chiral and twisted chiral superfields are likely to be intimately tied to the existence of mirror symmetry. Since we now know that suitable $N = 4$ parity twists exist, it is a natural question to wonder about $N = 4$ generalizations of mirror symmetry that might occur in some suitable systems.

Our observation regarding the $N = 4$ superstring "SU(2) singlet rule" likely has one other unsettling implication. Some time ago [11], a component level action was purportedly given for the 2D, $N = 4$ superstring. Following that, we asserted [12] the equivalence of our superspace construction in [9] to the prior work of Pernici and van Nieuwenhuizen. It now appears that our assertion was wrong. The work of ref. [11] does not describe one of the twisted hypermultiplets, the work in ref. [9] does.

In any event, the present work provides complete support for our interpretation that SM-I, SM-II and SM-III are distinct representations. This is particularly obvious in the case of SM-I versus the other two multiplets considered here. If the claim that all the multiplets in Table I are equivalent were true, then using field redefinitions a conformal theory could be turned into a non-conformal theory! We don't believe that even the most misguided "experts" would make such a claim. We thus end with the following canticle, "There are four distinct minimal (4,0) scalar multiplets."

"*Ye can lead a man up to the university but ye can't make him think.*"
– Finley Peter Dunne

Appendix: Results for Component Projection

In this appendix, we collect in one place the key results needed to derive our component results from our superspace ones.

- Covariant Derivatives:

  1. $\nabla_{\ddagger}| = \mathcal{D}_{\ddagger} + \psi_{\ddagger}{}^{+i}\partial_{+i} + \overline{\psi}_{\ddagger}{}^{+}{}_{i}\overline{\partial}_{+}{}^{i}$
  2. $\nabla_{=}| = \mathcal{D}_{=} + \psi_{=}{}^{+i}\partial_{+i} + \overline{\psi}_{=}{}^{+}{}_{i}\overline{\partial}_{+}{}^{i}$
  3. $\mathcal{D}_{\ddagger} = e_{\ddagger} + \omega_{\ddagger}\mathcal{M} + i\mathcal{A}_{\ddagger i}{}^{j}\mathcal{Y}_{j}{}^{i}$
  4. $\mathcal{D}_{=} = e_{=} + \omega_{=}\mathcal{M} + i\mathcal{A}_{= i}{}^{j}\mathcal{Y}_{j}{}^{i}$

- Spin Connections:

  1. $\omega_{\ddagger} = C_{\ddagger =}{}^{=}$
  2. $\omega_{=} = -C_{= \ddagger}{}^{\ddagger} + 2i\{\psi_{\ddagger}{}^{+i}\overline{\psi}_{=}{}^{+}{}_{i} - \psi_{=}{}^{+i}\overline{\psi}_{\ddagger}{}^{+}{}_{i}\}$

- Field Strengths:

  1. $-\frac{1}{2}\Sigma^{+i} = \mathcal{D}_{\ddagger}\psi_{=}{}^{+i} - \mathcal{D}_{=}\psi_{\ddagger}{}^{+i} + 2i\{\psi_{\ddagger}{}^{+j}\overline{\psi}_{=}{}^{+}{}_{j} - \psi_{=}{}^{+j}\overline{\psi}_{\ddagger}{}^{+}{}_{j}\}\psi_{\ddagger}{}^{+i}$
  2. $-\frac{1}{2}\overline{\Sigma}^{+}{}_{i} = \mathcal{D}_{\ddagger}\overline{\psi}_{=}{}^{+}{}_{i} - \mathcal{D}_{=}\overline{\psi}_{\ddagger}{}^{+}{}_{i} + 2i\{\psi_{\ddagger}{}^{+j}\overline{\psi}_{=}{}^{+}{}_{j} - \psi_{=}{}^{+j}\overline{\psi}_{\ddagger}{}^{+}{}_{j}\}\overline{\psi}_{\ddagger}{}^{+}{}_{i}$
  3. $-\frac{1}{2}\mathcal{R} = \mathcal{D}_{\ddagger}\omega_{=} - \mathcal{D}_{=}\omega_{\ddagger} - i\psi_{\ddagger}{}^{+i}\overline{\Sigma}^{+}{}_{i} - i\overline{\psi}_{\ddagger}{}^{+}{}_{i}\Sigma^{+i} + 2i\{\psi_{\ddagger}{}^{+i}\overline{\psi}_{=}{}^{+}{}_{i} - \psi_{=}{}^{+i}\overline{\psi}_{\ddagger}{}^{+}{}_{i}\}\omega_{\ddagger}$
  4. $-\frac{1}{2}\mathcal{F}_{i}{}^{j} = \mathcal{D}_{\ddagger}\mathcal{A}_{= i}{}^{j} - \mathcal{D}_{=}\mathcal{A}_{\ddagger i}{}^{j} + \{\psi_{\ddagger}{}^{+j}\overline{\Sigma}^{+}{}_{i} - \frac{1}{2}\delta_{i}{}^{j}\psi_{\ddagger}{}^{+k}\overline{\Sigma}^{+}{}_{k}\}$
     $- \{\overline{\psi}_{\ddagger}{}^{+}{}_{i}\Sigma^{+j} - \frac{1}{2}\delta_{i}{}^{j}\overline{\psi}_{\ddagger}{}^{+}{}_{k}\Sigma^{+k}\} \; + \; 2i\{\psi_{\ddagger}{}^{+k}\overline{\psi}_{=}{}^{+}{}_{k} - \psi_{=}{}^{+k}\overline{\psi}_{\ddagger}{}^{+}{}_{k}\}\mathcal{A}_{\ddagger i}{}^{j}$

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